1060, Fa 2013

Day 5 – Quadratics II & other common functions

Last Time:

we looked at how to graph something like

$$h(x) = 2(x+5)^2 + 3$$

this called the Standard form of the quadratic

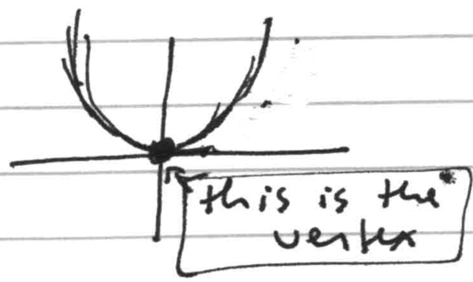
this is the graph of x^2

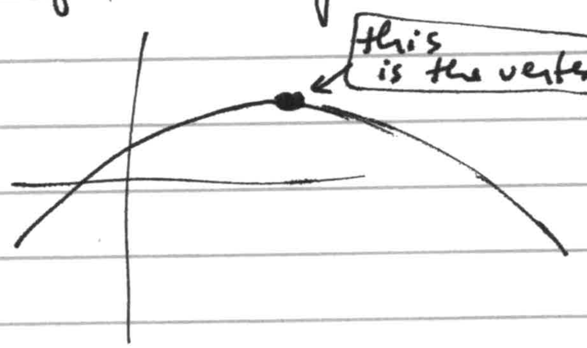
① stretched vertically by a factor of 2

② then moved left 5

③ then moved up 3

Define: the vertex of a parabola is the point at the top / bottom of the cup





Notice: the vertex is the

minimum point if the x^2 term is positive

and is the

maximum point

if the x^2 term is negative

To find the vertex

Notice the vertex of h(x) (above)

is what you get by moving

the vertex of x^2 the point (0,0)

left 5 & up 3

$\Rightarrow$ the vertex of

$h(x) = 2(x+5)^2+3$

is $(-5, 3)$

5-10

You can always find the vertex

from the standard form

by thinking this way.

~~And you can~~

You can always find the

standard form

by

completing the square

NOTE: the "old" way of completing the square Doesn't work well here!

This time: we'll complete the square for a trickier quadratic

Eg: Find the vertex of $h(x)$, and describe how to graph it

where

$$h(x) = 2x^2 + 8x + 6$$

factor 2 out of ALL terms

$$= 2\left[x^2 + 4x + 3\right]$$

now work INSIDE parenthesis

$2 \cdot 2 = 4$

$$= 2\left[x^2 + 4x + \underline{4} - \underline{4} + 3\right]$$

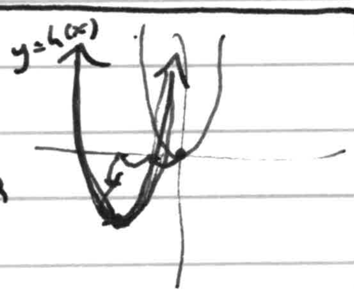
$\frac{4}{2} = 2$

$$= 2\left[(x+2)(x+2) - \underline{4} + 3\right]$$

$$= 2\left[(x+2)^2 - 1\right]$$

Remember to DISTRIBUTE the 2 to all terms

$$\boxed{h(x) = 2(x+2)^2 - 2}$$



so you get $h(x)$ by taking $2x^2$
moving left 2
& down 2

⇒ The vertex of $h(x)$ is $(-2,-2)$

2.2 - Other Common Functions

We can use the same tricks

[redacted] to graph

[redacted]

$y = f(x)$

$y = f(x-1)$ [redacted] moved right 1

"

5-10 $y = f(x) + 1$ [redacted] moved up 1

"

$y = -f(x)$ [redacted] reflected accross x-axis

and

"

$y = 2 \cdot f(x)$ [redacted] stretched vertically by a factor of 2

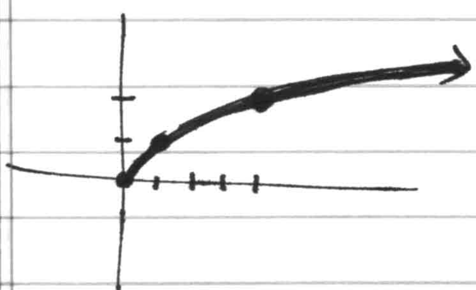
for other common functions

as long as we know the basic starting graph!

Square Root Functions

$f(x) = \sqrt{x}$

x	$f(x)$
-1	UNDEF'D
0	0
1	1
2	
4	2

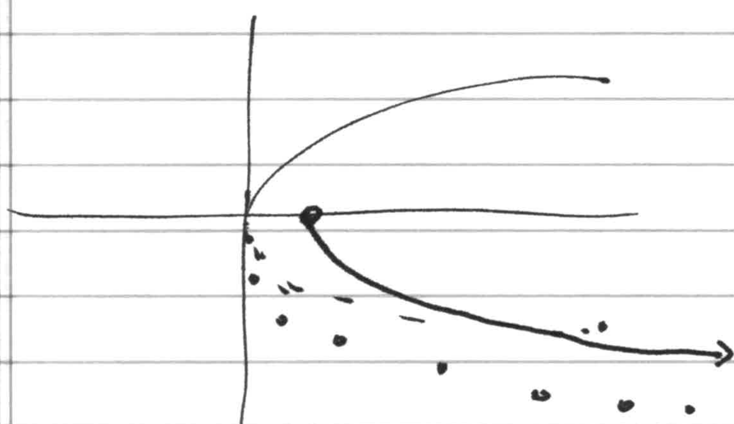


Domain of $\sqrt{x}$

is $x \geq 0$

is $[0, \infty)$

$f(x) = -2\sqrt{x-1}$



NOTICE Domain moves right 1

Domain of $-2\sqrt{x-1}$

is $x - 1 \geq 0$

is $x \geq 1$

is $[1, \infty)$

Day 7 2013 begins

Absolute Value Functions

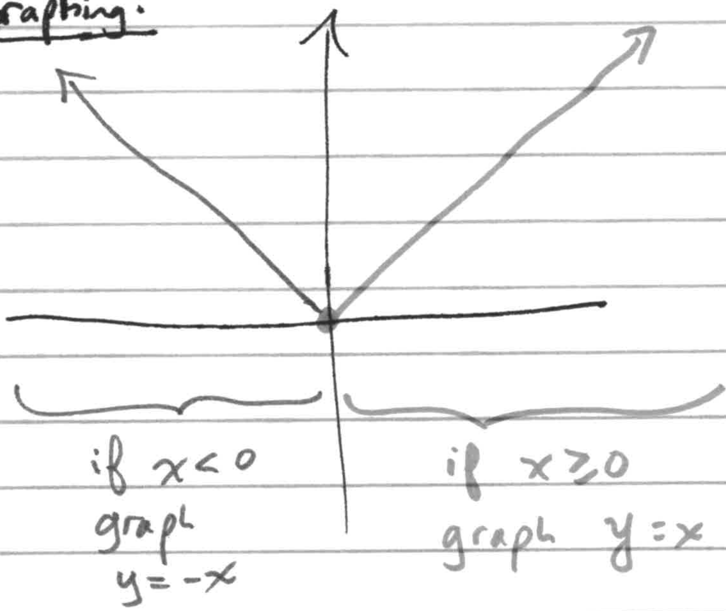
$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

means ~~is~~

if $x \geq 0$, then $|x| = x$

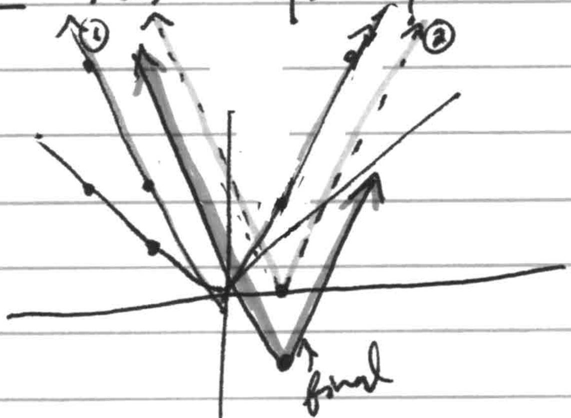
if $x < 0$, then $|x| = -x$

Graphing:



Can move this around as with the other examples.

Eg: $f(x) = 2|x-1| - 1$



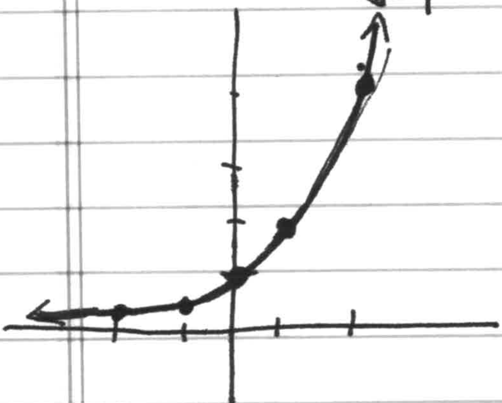
Domain: $(-\infty, \infty)$

Range: $[-1, \infty)$

Exponential Functions

eg: $f(x) = 2^x$

graph:



x	$f(x)$
-2	$2^{-2} = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$

NOTICE move right 1 Doubles the output

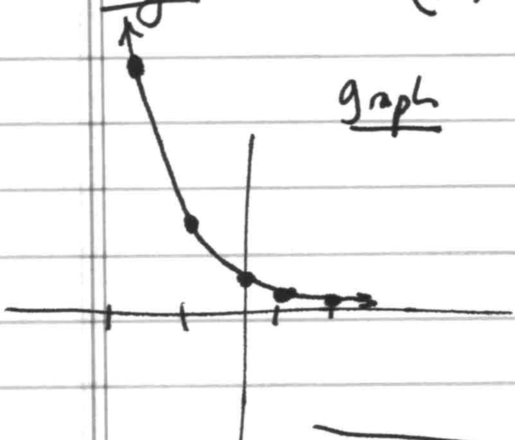
10 Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Doesn't even hit 0

eg: $f(x) = \left(\frac{1}{2}\right)^x$

graph



x	$f(x)$
-2	$\left(\frac{1}{2}\right)^{-2} = 4$
-1	$\left(\frac{1}{2}\right)^{-1} = 2$
0	$\left(\frac{1}{2}\right)^0 = 1$
1	$\left(\frac{1}{2}\right)^1 = \frac{1}{2}$
2	$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

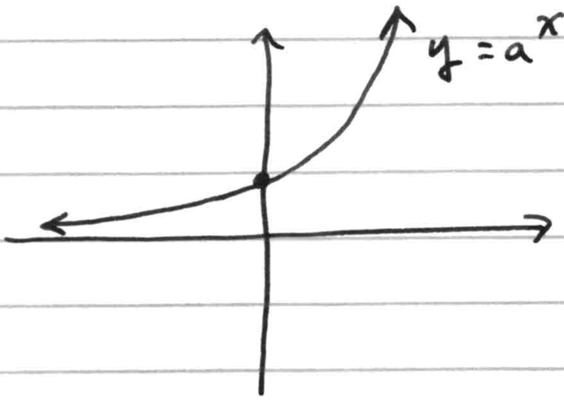
Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

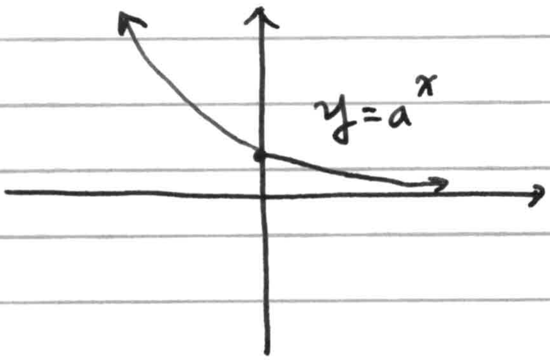
NOTICE moving right 1 HALVES the output

We can move these around As usual 2^x moved up 1 & left for other bases, Remember these Pictures:

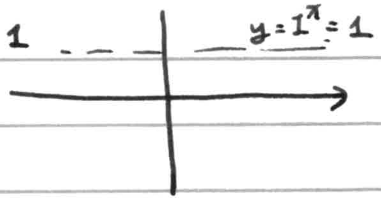
If $a > 1$:



If $0 < a < 1$:



[If $a = 1$



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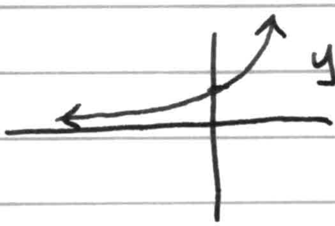
Let $e \approx 2.718$

this is an important # in calculus;



$y = e^x$ looks like this!